Науковий вісник НЛТУ України
Scientific Bulletin of UNFU
https://doi.org/10.36930/40330408
Article received 15.08.2023
Article accepted 31.08.2023
УДК 004.942:532.5
Correspondence author: M. V. Dendiuk dendyuk@nltu.edu.ua
ISSN 1994-7836 (print)
ISSN 2519-2477 (online)

O. V. Ovsiak, M. V. Dendiuk

Ukrainian National Forestry University, Lviv, Ukraine

MATHEMATICAL MODELING OF MOISTURE TRANSFER IN WOOD DRYING FOR THE TWO-DIMENSIONAL CASE

Mathematical modeling of moisture transfer in lumber during drying is a relevant and complex task with significant practical importance in the forestry and wood processing industries. Moisture transfer in lumber is a primary physical process during drying, and it directly affects the quality and efficiency of wood processing for various applications. Improving this process can lead to cost reduction and higher quality of the final product. However, predicting moisture release in lumber under different conditions and parameters remains a challenging task due to the complex nature of the process and various physical and mechanical factors that influence it. In total, this work dedicated to the development and utilization of mathematical models for the analysis and simulation of moisture transfer in lumber during drying. Special attention given to the use of finite element methods and cellular automata for modeling this process in a two-dimensional context. In this study, the cellular automata method employed as a potentially efficient tool for simulating moisture dynamics in lumber during drying. The fundamental idea of this method is to divide lumber into small entities or cells, each of which assigned its physical properties and state. Subsequently, the transfer of moisture between these cells simulated based on their states and surrounding conditions. This approach allows for a more detailed examination of moisture transfer processes and considers the influence of various factors, such as temperature, air humidity, lumber geometry, and its physical properties, on each individual cell. The results of the conducted research indicate that the cellular automata method proves to be an effective tool for modeling the dynamics of moisture transfer in lumber during drying. When comparing cellular automata with finite element methods, it is evident that cellular automata provide quicker results with fewer computational costs. This makes them an attractive choice for modeling complex processes like lumber drying and opens up possibilities for further research and innovations in this field.

Keywords: Anisotropy; finite element method; cellular automaton method; transition rules.

Introduction / Вступ

Setting the research task. In the field of forestry and wood processing, mathematical modeling of moisture transfer in lumber during drying is a relevant and one of the most important issues. This process directly affects the quality and production costs of finished products, and therefore precise mathematical modeling can significantly facilitate its management.

Characterization of the research problem in the global scientific literature. In the global scientific literature, the issue of moisture transfer in lumber during drying has already attracted the attention of recognized scientific experts. Researchers such as L. M. Hirnyk, Ya. I. Sokolovskyy, B. P. Pobereyko, V. O. Safarov, and many others have made a significant contribution to understanding this problem and have developed new or improved existing basic methods for modeling moisture transfer in lumber.

The relevance of the research. Despite the existing achievements, the problem of modeling moisture transfer in lumber remains complex and insufficiently studied. To achieve better results in lumber drying and enhance the efficiency of its processing, it is necessary to develop new approaches or refine existing methods for modeling. This need for further research underscores the significance of this study.

In summary, this work is dedicated to the development and utilization of mathematical models for the analysis and simulation of moisture transfer in lumber during drying. Special attention is given to the use of finite element methods and cellular automata for modeling this process in a two-dimensional context.

Object of research – the process of moisture transfer in lumber during their drying.

Subject of research – the mathematical modeling of this process.

The purpose of the work – develop a mathematical model for the analysis and modeling of moisture transfer in lumber during drying in a two-dimensional case.

To achieve this purpose, the following *main research objectives are identified*:

1. Review of moisture transfer process in wood under non-equilibrium conditions.
2. Improvement of the mathematical model for modeling moisture transfer in lumber during drying.
3. Solving a system of linear differential equations using cellular automata method.
4. Discussion and analysis of the obtained results.

Інформація про авторів:

Овсяк Олександр Володимирович, д-р техн. наук, доцент, кафедра інформаційних технологій. **Email:** ovsjak@nltu.edu.ua

Дендюк Михайло Володимирович, канд. техн. наук, доцент, кафедра інформаційних технологій. **Email:** dendyuk@nltu.edu.ua; https://orcid.org/0000-0002-7631-022X

Цитування за ДСТУ: Овсяк О. В., Дендюк М. В. Математичне моделювання вологоперенесення під час сушіння деревини для двовимірного випадку. Науковий вісник НЛТУ України. 2023, т. 33, № 4. С. 59–64.

Citation APA: Ovsiak, O. V., & Dendiuk, M. V. (2023). Mathematical modeling of moisture transfer in wood drying for the two-dimensional case. *Scientific Bulletin of UNFU*, 33(4), 59–64. https://doi.org/10.36930/40330408

Analysis of recent research and publications. During the analysis of recent research and publications in this field, the following can be highlighted: In paper [10], authors constructed a two-dimensional mathematical model of the process of convective drying of anisotropic porous materials, considering the movement of the phase transition boundary. In paper [6], a three-dimensional mathematical model of heat and moisture transfer in wood during convective drying was developed. Authors took into account the influence of the surrounding fluid flow and described the characteristics of both wood and fluid flow using conservation equations of mass, momentum, energy, as well as thermodynamic relations. In turn, article [7] focused on a mathematical model of moisture transfer during the drying of Pinus massoniana wood. The work allowed for simulating moisture distribution inside the wood and proposing an empirical formula for calculating wood permeability. In paper [12], a mathematical model of heat and mass distribution in capillary-porous materials with fractal structure during wood drying is investigated. Fractional partial differential equations are used to describe this model. In addition, in paper [5], a mathematical model of moisture transfer during Masson pine wood drying is discussed. The work divides the drying process into two phases and provides information about moisture distribution inside the wood. In turn in paper [8], a mathematical model for heat and moisture transfer in porous media, taking into account internal heat and moisture release, is developed. At the last paper [11], a mathematical model of heat and moisture transfer in materials with fractal structure, considering memory and spatial correlation effects, is synthesized. The work includes the development of numerical methods for solving mathematical models and proposes algorithm parallelization.

Despite the extensive research in the field of moisture transfer modeling in wood, it is worth noting that the authors of these studies did not attempt to utilize cellular automata for this purpose. Thus, this work explores the possibility of using cellular automata for modeling moisture transfer in wood.

Research results and their discussion / Результати дослідження та їх обговорення

Overview of the non-stationary process of moisture transfer in wood. The non-stationary process of moisture transfer in the process of drying wood at the stage of removal of bound moisture described by equation [3] in partial derivatives:

$$\frac{\partial U}{\partial \tau} = \frac{\partial}{\partial x}\left(\alpha_x \frac{\partial U}{\partial x}\right) + \frac{\partial}{\partial y}\left(\alpha_y \frac{\partial U}{\partial y}\right) + \frac{\partial}{\partial z}\left(\alpha_z \frac{\partial U}{\partial z}\right). \quad (1)$$

At the same time, it has boundary conditions of the 3rd kind:

$$U = U_B(S), \quad (2)$$

$$\alpha \frac{\partial U}{\partial n} = \beta\left(U_\Pi - U_R\right). \quad (3)$$

In addition, an initial condition:

$$U\big|_{\tau=0} = U_0, \quad (4)$$

where: U is moisture content, kg/kg; τ is time, s; x, y, z are coordinate of axes; α_x, α_y, α_z are moisture conductivity coefficients along the anisotropy axes, m^2/s; $U_B(S)$ is the moisture content on the surface, which can be a function of coordinates; n is external normal; β is moisture exchange coefficient, m^2/s; U_Π is moisture content on the surface; U_R is the equilibrium moisture content, which is a function of the temperature t_c and the relative humidity of the external environment; U_0 is the initial humidity distribution.

To solve equation (1) with boundary conditions (2), (3) and initial condition (4), we place the Cartesian coordinate system for lumber as follows. Radial direction across the fibers along the trunk radius (thickness b) – y axis, tangential direction across the fibers tangent to the border between annual rings (width a) – x axis, axial direction along the fibers (length L) – z axis (Figure 1).

Moisture removed from the wood in all directions, but since the cross-sectional dimensions of most lumber to be dried are small compared to their length, the coefficient of moisture conductivity along the grain is much greater than the coefficients across the grain. The moisture difference along the grain will be also negligible, that is $\partial^2 U / \partial z^2 \approx 0$, and therefore lumber can be considered as a two-dimensional body.

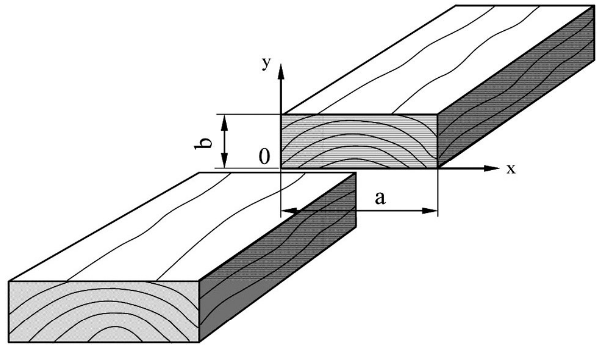

Fig. 1. Cross section of lumber / Поперечний переріз пиломатеріалів

So, for the cross-section of lumber, which is located in a region sufficiently far from the ends with constant coefficients of moisture conductivity, equation (1) can be rewritten as follows:

$$\frac{\partial U}{\partial \tau} = \alpha_x \frac{\partial^2 U}{\partial x^2} + \alpha_y \frac{\partial^2 U}{\partial y^2}, \quad (5)$$

with boundary (2), (3) and initial (4) conditions.

Modeling of moisture transfer process in drying wood. To modeling the moisture transfer process during wood drying using the cellular automata method [13], at first we considered the possibility of using the finite element method. The main idea of this method is that any continuous quantity, such as moisture content, can be approximated using a discrete model. This discrete model built on a set of functions defined on a limited number of elements [9]. Such functions defined using the values of a continuous quantity in a limited number of points of the domain. To build a discrete model, the domain divided into a limited number of nodal points. The domain of the definition of a continuous value also divided into a limited number of elements that generally approximate the distribution of moisture in the cross section of lumber. A continuous quantity is approximated on each element by a polynomial determined using the nodal values of this quantity.

From the variation point of view, the solution of equation (5) with boundary conditions (2) and (3) is equivalent to finding the minimum of the next functional:

$$\chi = \int_V \frac{1}{2}\left[\alpha_x\left(\frac{\partial U}{\partial x}\right)^2 + \alpha_y\left(\frac{\partial U}{\partial y}\right)^2 + 2\frac{\partial U}{\partial \tau}U\right]dV + \int_{S_2} \frac{\beta}{2}\left(U - U_R\right)^2 dS. \quad (6)$$

Before calculating the integrals, let us make a substitution in the functional (6) by entering the following matrices:

$$\{g\}^T = \begin{bmatrix} \frac{\partial U}{\partial x} & \frac{\partial U}{\partial y} \end{bmatrix} \tag{7}$$

and

$$DU = \begin{bmatrix} \alpha_x & 0 \\ 0 & \alpha_y \end{bmatrix}. \tag{8}$$

Then let us write it in the next form:

$$\chi = \int_V \frac{1}{2}\Big[\{g\}^T[DU]\{g\}\Big]dV + \int_V \frac{\partial U}{\partial \tau} U\,dV + \\ + \int_{S_2} \frac{1}{2}\beta\Big[U_{\Pi}^2 - 2U_{\Pi}U_R + U_R^2\Big]dS. \tag{9}$$

It is clear that the integrals in the functional (9) must be broken down into integrals for individual elements, which give us the following notation:

$$\chi = \sum_{e=1}^{E}\Bigg[\int_{V^{(e)}} \frac{1}{2}\Big\{g^{(e)}\Big\}^T\Big[DU^{(e)}\Big]\Big\{g^{(e)}\Big\}dV + \int_V \frac{\partial U^{(e)}}{\partial \tau}U^{(e)}dV + \\ + \int_{S_2^{(e)}} \frac{\beta^{(e)}}{2}U^{(e)}U^{(e)}dS - \int_{S_2^{(e)}} \beta^{(e)}U^{(e)}U_R dS + \int_{S_2^{(e)}} \frac{\beta^{(e)}}{2}U_R^2 dS\Bigg], \tag{10}$$

where E is the total number of elements, and the superscript (e) means an arbitrary element.

Thus, the contribution of a separate element $\chi^{(e)}$ to the functional χ will be:

$$\chi = \chi^{(1)} + \chi^{(2)} + \ldots + \chi^{(E)} = \sum_{e=1}^{E}\chi^{(e)}. \tag{11}$$

In the general, the value of the moisture content approximated by the dependence:

$$U^{(e)} = \Big[N^{(e)}\Big]\cdot\{U\}, \tag{12}$$

where: $[N^{(e)}]$ are shape functions of the selected element; $\{U\}$ is column matrix of nodal values of moisture content.

To find the value $\{g\}$ we substitute the expression for approximating the moisture content (12) into formula (7) and obtain:

$$\Big\{g^{(e)}\Big\} = \begin{Bmatrix} \frac{\partial U^{(e)}}{dx} \\ \frac{\partial U^{(e)}}{dy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial N_1^{(e)}}{dx} & \frac{\partial N_2^{(e)}}{dx} & \cdots & \frac{\partial N_p^{(e)}}{dx} \\ \frac{\partial N_1^{(e)}}{dy} & \frac{\partial N_2^{(e)}}{dx} & \cdots & \frac{\partial N_p^{(e)}}{dx} \end{bmatrix}\begin{Bmatrix} U_1 \\ U_2 \\ \vdots \\ U_P \end{Bmatrix}, \tag{13}$$

or

$$\Big\{g^{(e)}\Big\} = \Big[BU^{(e)}\Big]\cdot\{U\}, \tag{14}$$

where: p is the number of element nodes; $[BU]$ is matrix of derivatives of the shape function of the selected element.

Therefore, the functional for a single element $\chi^{(e)}$ can be written as follows:

$$\chi^{(e)} = \int_{V^{(e)}} \frac{1}{2}\{U\}^T\Big[BU^{(e)}\Big]^T\Big[DU^{(e)}\Big]\Big[BU^{(e)}\Big]\{U\}dV + \\ + \int_{V^{(e)}} \Big[N^{(e)}\Big]\{U\}\Big[N^{(e)}\Big]\frac{\partial\{U\}}{\partial\tau}dV + \\ + \int_{S_2^{(e)}} \frac{\beta}{2}\{U\}^T\Big[N^{(e)}\Big]^T\Big[N^{(e)}\Big]\{U\}dS - \\ - \int_{S_2^{(e)}} \beta U_R\Big[N^{(e)}\Big]\{U\}dS + \int_{S_2^{(e)}} \frac{\beta}{2}U_U^2\,dS. \tag{15}$$

Minimization of the functional χ requires the fulfillment of such a relation:

$$\frac{\partial\chi}{\partial\{U\}} = \frac{\partial}{\partial\{U\}}\sum_{e=1}^{E}\chi^{(e)} = \sum_{e=1}^{E}\frac{\partial\chi^{(e)}}{\partial\{U\}} = 0, \tag{16}$$

where $\{U\}$ are nodal values of moisture content.

Next, we differentiate the terms of the functional for each element (15) by the nodal values of the moisture content $\{U\}$:

$$\frac{\partial}{\partial\{U\}}\int_{V^{(e)}} \frac{1}{2}\{U\}^T\Big[BU^{(e)}\Big]^T\Big[DU^{(e)}\Big]\Big[BU^{(e)}\Big]\{U\}dV = \\ = \int_{V^{(e)}} \Big[BU^{(e)}\Big]^T\Big[DU^{(e)}\Big]\Big[BU^{(e)}\Big]\{U\}dV, \tag{17}$$

$$\frac{\partial}{\partial\{U\}}\int_{V^{(e)}} \Big[N^{(e)}\Big]\{U\}\Big[N^{(e)}\Big]\frac{\partial\{U\}}{\partial\tau}dV = \\ = \int_{V^{(e)}} \Big[N^{(e)}\Big]^T\Big[N^{(e)}\Big]dV\frac{\partial\{U\}}{\partial\tau}, \tag{18}$$

$$\frac{\partial}{\partial\{U\}}\int_{S_2^{(e)}} \frac{\beta}{2}\{U\}^T\Big[N^{(e)}\Big]^T\Big[N^{(e)}\Big]\{U\}dS = \\ = \int_{S_2^{(e)}} \beta\Big[N^{(e)}\Big]^T\Big[N^{(e)}\Big]\{U\}dS, \tag{19}$$

$$\frac{\partial}{\partial\{U\}}\int_{S_2^{(e)}} \beta U_R\Big[N^{(e)}\Big]\{U\}dS = \int_{S_2^{(e)}} \beta U_R\Big[N^{(e)}\Big]^T dS, \tag{20}$$

$$\frac{\partial}{\partial\{U\}}\int_{S_2^{(e)}} \frac{\beta}{2}U_R^2\,dS = 0. \tag{21}$$

The contribution of a separate element $\partial\chi^{(e)}/\partial\{U\}$ to the total amount $\partial\chi/\partial\{U\}$ will be as follows:

$$\frac{\partial\chi^{(e)}}{\partial\{U\}} = \Bigg(\int_{V^{(e)}} \Big[BU^{(e)}\Big]^T\Big[DU^{(e)}\Big]\Big[BU^{(e)}\Big]dV + \\ + \int_{S_2^{(e)}} \beta\Big[N^{(e)}\Big]^T\Big[N^{(e)}\Big]dS\Bigg)\{U\} + \\ + \int_{V^{(e)}} \Big[N^{(e)}\Big]^T\Big[N^{(e)}\Big]dV\frac{\partial\{U\}}{\partial\tau} - \int_{S_2^{(e)}} \beta U_R\Big[N^{(e)}\Big]^T dS. \tag{22}$$

Marking:

$$\Big[ku^{(e)}\Big] = \int_{V^{(e)}} \Big[BU^{(e)}\Big]^T\Big[DU^{(e)}\Big]\Big[BU^{(e)}\Big]dV + \\ + \int_{S_2^{(e)}} \beta\Big[N^{(e)}\Big]^T\Big[N^{(e)}\Big]dS, \tag{23}$$

$$\Big[cu^{(e)}\Big] = \int_V [N]^T[N]\,dV, \tag{24}$$

$$\Big\{fu^{(e)}\Big\} = -\int_{S_2^{(e)}} \beta U_R\Big[N^{(e)}\Big]^T dS, \tag{25}$$

equation (22) write in the next matrix form:

$$\frac{\partial\chi^{(e)}}{\partial\{U\}} = \Big[ku^{(e)}\Big]\{U\} + \Big[cu^{(e)}\Big]\frac{\partial\{U\}}{\partial\tau} + \Big\{fu^{(e)}\Big\}. \tag{26}$$

If we consider equation (26), we can write equation (16) in the next form:

$$\frac{\partial\chi}{\partial\{U\}} = \sum_{e=1}^{E}\Bigg(\Big[ku^{(e)}\Big]\{U\} + \Big[cu^{(e)}\Big]\frac{\partial\{U\}}{\partial\tau} + \Big\{fu^{(e)}\Big\}\Bigg) = 0, \tag{27}$$

or

$$[CU]\frac{\partial\{U\}}{\partial\tau} + [KU]\{U\} + \{FU\} = 0, \tag{28}$$

where: $[KU]$ is moisture conductivity matrix; $[CU]$ is damping matrix; $\{FU\}$ is the load matrix.

All integrals in equations (23)-(25) taken by individual elements. Summing up the contributions of individual elements carried out by the method of direct stiffness.

Solving the system of linear differential equations by the method of cellular automata. To obtain the value of the moisture content $\{U\}$ at each point of the time interval, it is necessary to solve the system of linear differential equations of the first order (28), for the solution of which was used different methods. At the same time, the partial time derivative replaced by its finite-difference analogue using the central difference scheme, or the Galerkin method. This method also used in combination with finite elements in the time domain, not in the spatial domain [9]. However, we tried to apply a new and faster method such us cellular automata [1].

Regardless of the method of solving equation (28), at the end, the system of equations for determining the moisture content in wood during drying, taking into account the anisotropy of properties, will have the next form [2]:

$$[A]\{U\}_{NEXT} = \{R\}, \tag{29}$$

where:

$$[A] = [KU] + \frac{2}{\Delta\tau}[CU], \tag{30}$$

$$\{R\} = \left(\frac{2}{\Delta\tau}[CU] - [KU]\right)\{U\}_{LAST} + 2\{FU\}. \tag{31}$$

Matrix $[A]$ is a combination of matrices $[CU]$ and $[KU]$ and depends on the time step $\Delta\tau$. If $\Delta\tau$ and the material parameters don't depend on time or on $\{U\}$, then the matrix $[A]$ will be the same at all time points. If $\Delta\tau$ or the material parameters change during the solution, then the matrix $[A]$ must be recalculated each time, summing over all elements and then triangulation. This procedure significantly increases the volume of calculations, but it is necessary when $\Delta\tau$ is variable or in the case where the moisture transfer coefficients α_x, α_y are functions of moisture, etc.

In general, the process of creating a cellular automata model [4] and transition rules, which based on the system of equations (29)-(31) with the representation of the matrix $[A]$ as a combination of the matrices $[CU]$ and $[KU]$, described in three stages:

Stage 1. Creation of a cellular automata model:

This stage consists of several actions, in particular: at first, we must select the geometry of the study area, where the moisture transfer process takes place. In this case, it is one lumber, which mentioned earlier (look Figure 1).

Next, it is necessary to divide the studied area into cells. At the same time, each cell represents a discrete object for which the state of humidity will be determined at different moments of time. Since our studied area has a simple geometric shape, the process of partitioning is not complicated. The only difficulty at this stage is to choose the correct cell size, since each of them must have the same dimensions and together they must clearly cover the studied area, both in height (parameter $b - Y$ axis) and in width (parameter $a - X$ axis).

At the end of this stage, it is necessary to determine the cell parameters. The set of these parameters will be the same for all cells, but their values will differ. In general, material parameters that directly affect moisture transfer and other physical and mechanical properties added here.

Stage 2. Development of transition rules:

In general, several basic transition rules were developed, the implementation of which makes it possible to calculate new values of moisture content in the material over time. Therefore, the developed transition rules have the following form:

- For each cell, we calculate the matrices $[CU]$ and $[KU]$ based on the parameters of the material and humidity $\{U\}$ according to equations (30) and (31).
- Calculate the matrix $[A]$ as a combination of the matrices $[CU]$ and $[KU]$, with taking into account the time step $\Delta\tau$.
- We set the initial conditions for the $\{U\}_{LAST}$ and the load $\{FU\}$.
- For each time step, we calculate the vector $\{R\}$ based on the values $\{U\}_{NEXT}$ and $\{FU\}$. At the same time, the values of the matrices $[CU]$, $[KU]$ and $[A]$ are calculated according to equations (30) and (31).
- Find the new value of $\{U\}_{NEXT}$ according to equation (29).

Stage 3. Iterative process:

This step involves repeating step 2 until the desired accuracy, specified time interval, or equilibrium state is achieved. The last corresponds to the situation when the humidity values in the cells no longer change over time or change slightly, thereby reaching a stable level. In this state of equilibrium, the studied system can be stable, and the dynamics of the processes can be coordinated. To reach an equilibrium state, the iterative process performed until the stability or accuracy condition is satisfied.

Thus, in the process of creating a cellular automata model and transition rules, we use the representation of the matrix $[A]$ as a combination of the matrices $[CU]$ and $[KU]$, which depend on the time step $\Delta\tau$. They can also change during the solution process depending on the parameters material and humidity $\{U\}$. This approach makes it possible to more accurately take into account changes in humidity in the material during the process of moisture transfer. In turn, the use of the iterative process makes it possible to achieve a state of equilibrium, thereby obtaining more accurate results of solving a given system of equations.

Based on the mathematical model (29)-(31), we developed the software in accordance with the principles of object-oriented programming. This approach allow us to describe the object and the actions on it as a single entity, providing the opportunity to divide a complex problem into a number of simpler ones. In general, the software consists of a main module and two classes: "Point_class" and "Element", and its class diagram you can see in Figure 2.

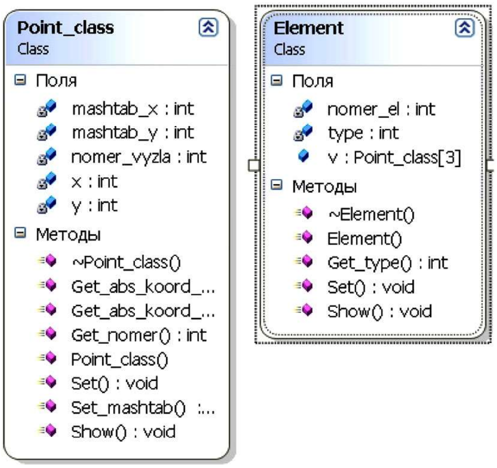

Fig. 2. Class diagram of the developed software / Діаграма класів розробленого програмного забезпечення

During the execution of the task, we created a set of objects of the "Point_class" class. These objects store the coordinates of the points that were determined when using the finite element method, or the cell number, if we used the cellular automata method. After that, we formed a collection of objects of the "Element" class. Each object of this class contains information about three points of the "Point_class" class or the type and number of the corresponding cell on the cellular automata field.

In general, the software implemented in console mode in order to optimize the use of computer hardware resources and, as a result, increase productivity. This becomes especially relevant even in the conditions of modern development of computer technology. In the course of research, it we established that dividing the problem domain into 2,000 elements or cells allows calculations to perform for one iteration in about 30-40 seconds on average.

Analysis of the obtained results. Determination of non-stationary distribution of moisture transfer in the process of wood drying was carried out for pine (ρ_0 = 400 kg/m^3) with initial values of moisture W_0 = 0.3 % and temperature t_0 = 20 °C with the following conditions of the drying process: t = 79 °C, φ = 77 %, v = 2 m/s (φ, v – relative humidity and drying agent speed).

As a result of calculations using the finite element method and the cellular automata method, several surface graphs were obtained. As a surface, we decided to take a cross-section of lumber along its center, as shown in Figure 1. We used the Python language to construct these two graphs, and their appearance you can see in Figure 3.

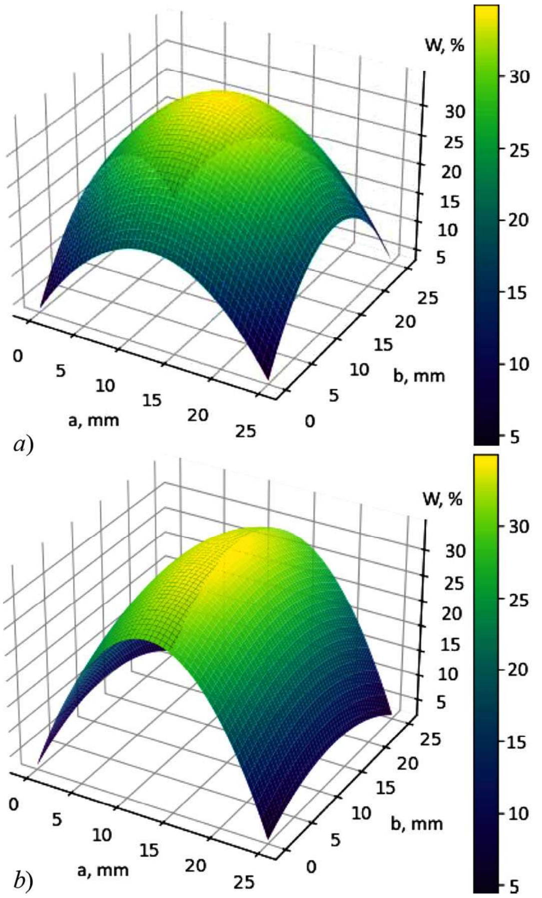

Fig. 3. Distribution of moisture in a section of pine lumber with a size of 25×25 mm using the finite element method (left) and the cellular automata method (right) / Розподіл вологи в розрізі соснових пиломатеріалів розміром 25×25 мм методом скінченних елементів (ліворуч) та методом клітинних автоматів (праворуч)

According to this figure, we see that both methods showed adequate results. However, when we comparing them, some discrepancies are immediately noticeable at the edges of the cross-sectional surface when we using the cellular automata method. This may be due to an excess or insufficient number of transition rules or cell parameters. Despite this, the value of moisture content in the middle of the material coincides with the results obtained using the finite element method by approximate 80 %. However, we spent less time to obtain these values. Despite this, we are confident that significantly better results can be achieve by applying a combination of different cell parameters and possibly transition rules. Considering this, the research will persist.

Discussion of research results. The results of this research provide valuable insights into the mathematical modelling of moisture transfer in lumber during drying. Comparing the obtained results with similar studies in this field [5, 6, 7, 8, 10, 11, 12], it becomes evident that cellular automata offer a new and promising approach for modelling moisture dynamics in wood. In contrast to the conventional finite element methods used in previous research, the use of cellular automata has proven to be more efficient, providing quicker results with reduced computational costs. This study demonstrates the potential of this approach and opens up pathways for further research and innovations in this field, potentially leading to more effective modelling of wood drying processes.

So, based on the results of the work performed, it is possible to formulate the following scientific novelty and practical significance of the research results.

Scientific novelty of the obtained research results – development of a cellular automata model for simulating moisture transfer in lumber during drying, taking into account anisotropy.

Practical significance of the research results – a new approach has been developed that offers a more efficient and cost-effective way of modelling and analysing the process of drying lumber.

Conclusions / Висновки

This study conducted an extensive review of the non-stationary process of moisture transfer in lumber and explored two distinct methods, namely the finite element method and the cellular automata method, for modeling this complex process. The research objectives encompassed several key aspects.

The first objective involved a comprehensive overview of moisture transfer in lumber under non-equilibrium conditions, serving as the foundational step for subsequent modeling. The development and refinement of a mathematical model to simulate moisture transfer in lumber during drying constituted the second objective, aiming to enhance modeling accuracy.

The third objective centered on solving a system of linear differential equations using the cellular automata method, presenting an innovative alternative to traditional finite element methods. Finally, the obtained results underwent rigorous discussion and analysis, comparing the outcomes of both modeling methods.

While both approaches demonstrated adequate results, the cellular automata method displayed promise in terms of computational efficiency and accuracy in specific areas. The comparative analysis did reveal minor discrepancies at the cross-sectional surface edges of lumber when employing the cellular automata method. Future research will prioritize refining transition rules and cell parameters to address these discrepancies and further enhance the model's precision.

At the end, the combination of diverse cell parameters and refined transition rules will yield a more accurate and efficient modeling approach for moisture transfer in lumber. This study contributes valuable insights to the field of moisture transfer modeling in lumber, offering potential enhancements for the wood processing industries.

References

1. Chopard, B., & Droz, M. (1998). Cellular automata modeling of physical systems. *Cambridge University Press*. https://doi.org/10.1017/CBO9780511549755
2. Dendyuk, M. V., Pobereyko, B. P., & Sokolovskiy, Ya. I. (2003). Application of the finite element method for calculating nonstationary moisture transfer fields in drying wood. *Forestry, Wood and Paper Industry*, 28, 100–106. [In Ukrainian].
3. Hirnyk, L. M., et al. (1993). *Mathematical modeling of convective drying processes*. Kyiv: Budivelnyk. [In Ukrainian].
4. Ilachinski, A. (2001). Cellular Automata: A Discrete Universe. *World Scientific Publishing Co Pte Ltd.* https://doi.org/10.1142/4702
5. Jie, Zhang, Ping Miao, Di Zhong, & Lin Liu. (2014). Mathematical modeling of drying of Masson pine lumber and its asymmetrical moisture content profile. *Holzforschung*, 68(3), 313–321. https://doi.org/10.1515/hf-2013-0077
6. Jingyao, Zhao, & Yingchun, Cai. (2017). A comprehensive mathematical model of heat and moisture transfer for wood convective drying. *Holzforschung*, 71(5), 425–435. https://doi.org/10.1515/hf-2016-0148
7. Krabbenhøft, Kristian. (2004). Moisture Transport in Wood: A Study of Physical-Mathematical Models and their Numerical Implementation. Kgs. Lyngby: Technical University of Denmark.
8. Ravshanov, N., Shadmanov, I., Kubyashev, K., & Khikmatullaev, S. (2021). Mathematical modeling and research of heat and moisture transfer processes in porous media. *Construction Mechanics, Hydraulics and Water Resources Engineering. International Scientific Conference*, 1–19. https://doi.org/10.1051/e3sconf/202126401038
9. Sokolovskiy, Ya. I., & Bakalec, A. V. (2004). Calculation of Anisotropic Nonstationary Temperature-Moisture Fields in Drying Wood Using Finite Element Method. *Forestry, Wood and Paper Industry*, 29, 109–117. [In Ukrainian].
10. Sokolovskyy, Ya., Boretska, I., Gayvas, B., & Kroshnyy, I. (2018). Mathematical Modeling of the Heat-Mass-Exchange in Anisotropic Environments Taking into Account the Boundary of Phase Transition. 13th *International Scientific and Technical Conference on Computer Sciences and Information Technologies* (CSIT), Lviv, Ukraine, 147–150. https://doi.org/10.1109/STC-CSIT.2018.8526712
11. Sokolovskyy, Ya., Shymanskyi, V., Levkovych, M., & Yarkun, V. (2016). Mathematical modeling of heat and moisture transfer and reological behavior in materials with fractal structure using the parallelization of predictor-corrector numerical method. IEEE *First International Conference on Data Stream Mining & Processing* (DSMP), Lviv, Ukraine, 108–111. https://doi.org/10.1109/DSMP.2016.7583518
12. Sokolowskyi, Ya., & Shymanskyi, V. (2014). Mathematical Modelling of Non-Isothermal Moisture Transfer and Rheological Behavior in Cappilary-Porous Materials with Fractal Structure During Drying. *Computer and Information Science*, 7(4), 111–122. https://doi.org/10.5539/cis.v7n4p111
13. Wolfram, S. (1983). Statistical mechanics of cellular automata. *Reviews of Modern Physics*, 55(3), 601–644, https://doi.org/10.1103/RevModPhys.55.601

О. В. Овсяк, М. В. Дендюк

Національний лісотехнічний університет України, м. Львів, Україна

МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ВОЛОГОПЕРЕНЕСЕННЯ ПІД ЧАС СУШІННЯ ДЕРЕВИНИ ДЛЯ ДВОВИМІРНОГО ВИПАДКУ

Математичне моделювання вологоперенесення в пиломатеріалах під час сушіння є актуальним і складним завданням, яке має велике практичне значення в деревообробній галузі. Вологоперенесення є основним фізичним процесом під час сушіння, від нього залежить якість та ефективність оброблення пиломатеріалів для їх подальшого використання в різних сферах. Удосконалення цього процесу може сприяти зменшенню витрат та підвищенню якості остаточного продукту. Проте прогнозування вологовіддачі в пиломатеріалах за різних умов і параметрів є складним завданням через вплив низки фізичних і механічних чинників. У цій роботі розроблено та застосовано математичні моделі для аналізу та моделювання процесу вологоперенесення в пиломатеріалах під час сушіння. Особливу увагу приділено використанню методів скінченних елементів і клітинних автоматів для моделювання цього процесу в двовимірному випадку. Використано метод клітинних автоматів як потенційно ефективний інструмент для моделювання динаміки вологи в пиломатеріалах під час сушіння. Основна ідея цього методу полягає в тому, що пиломатеріал поділяють на маленькі осередки, тобто клітини, і для кожної з них визначають її фізичні властивості та стан. Потім імітують передачу вологи між цими клітинами залежно від їхнього стану та навколишніх умов. Цей підхід дає змогу дещо детальніше проаналізувати процес вологоперенесення та враховувати вплив різних чинників, таких як температура, вологість повітря, геометрія пиломатеріалу та її фізичні властивості на кожну клітину окремо. За отриманими результатами дослідження з'ясовано, що метод клітинних автоматів є ефективним інструментом для моделювання динаміки вологоперенесення в пиломатеріалах під час сушіння. Порівнюючи метод клітинних автоматів з методом скінченних елементів встановлено, що клітинні автомати дають змогу отримувати результати швидше та з меншими обчислювальними витратами. Це робить їх привабливим вибором для моделювання складних процесів, таких як сушіння пиломатеріалів, а також дає перспективи для подальших досліджень та інновацій у цій сфері людської діяльності.

Ключові слова: анізотропія; метод скінченних елементів; метод клітинних автоматів; правила переходів.