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# ТЕОРЕТИЧНЕ ТА ЕКСПЕРИМЕНТАЛЬНЕ ВИЗНАЧЕННЯ ГРАНИЦЬ ДЕМФУВАННЯ У ШАРУВАТИХ СТІЛЬНИКОВИХ СТРУКТУРАХ

Розглядаються теоретичні моделі для напружено-деформованого стану та демфування у неоднорідних тонкостінних структурах. Для усталених вібраційних станів визначені демфуючі властивості. Для порівняння з вібраційними дослідженнями отримано континуальні розрахункові моделі.

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# Theoretical and experimental damping bounds predictions for sandwich honeycomb structures

Theoretical models for the stress-strain state and the damping in non-uniform thin-walled structures are discussed. The damping properties by steady-state vibration are established. Continuous system models are obtained for comparison with vibration test.

## Introduction

The multifunctional properties of cellular solids have generated great interest for their application in ultra-light structures. The properties that appear most attractive are those that govern the use of cellular solids as cores for panels and shells having lower weight than competing materials, and potentially superior heat dissipation, vibration control and energy dissipation characteristics. Commercially available cellular solids and foams have random microstructures and their properties have been thoroughly documented. In this paper, the dynamic response and the sound radiation of sandwich beams is investigated.

The reduction of vibration responses and the transmission of vibratory energy in layered composite sandwich structures and mechanical systems has been the subject of investigation for many years. Many numerical schemes (NS) have been developed [1-6]. Most of them are based on the Tymoshenko theory. In such studies many factors are neglected, such as the normal strains in the materials, inhomogeneities of stress-strain state in joints and the presence of holes and inclusions. The neglect of these factors may have important consequences for high frequency excitation. Now on the basis of refined thin-walled element theory, we have tried to determine the stresses and energy dissipation.

## 1. Loss factor prediction for honeycomb sandwich structure

Consider now honeycomb sandwich structure (Fig. 1)

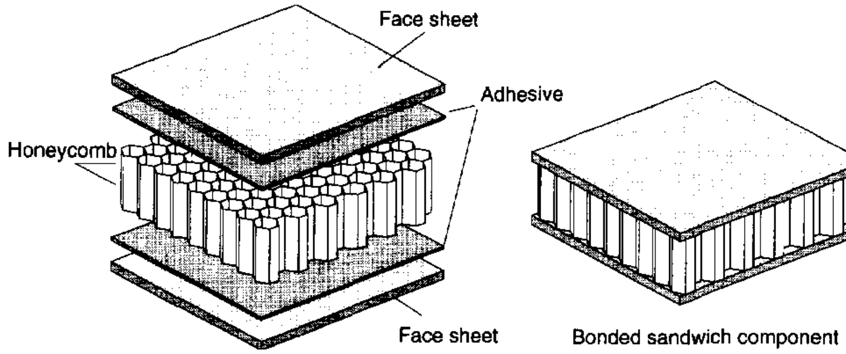


***Fig. 1. Schematic of adhesive bonding of sandwich structures***

The energy method is used only for the real part of the effective complex coefficients. For a laminate plate, from the energy method [6, 7], the total damping loss factor is predicted as:

$$\frac{E_d}{E} = \frac{\sum\limits_{i=1,N} \eta_i \int\limits_{V_i} (\sigma_{xx}\varepsilon_{xx} + \sigma_{zz}\varepsilon_{zz} + \tau_{xz}\varepsilon_{xz} dV = 0}{\sum\limits_{i=1,N} \int\limits_{V_i} (\sigma_{xx}\varepsilon_{xx} + \sigma_{zz}\varepsilon_{zz} + \tau_{xz}\varepsilon_{xz} dV = 0} \quad . \tag{1}$$

In Fig. 2 the loss factors are presented, based on refined dynamic theory of plate vibration [5-7] and Eq.(1)

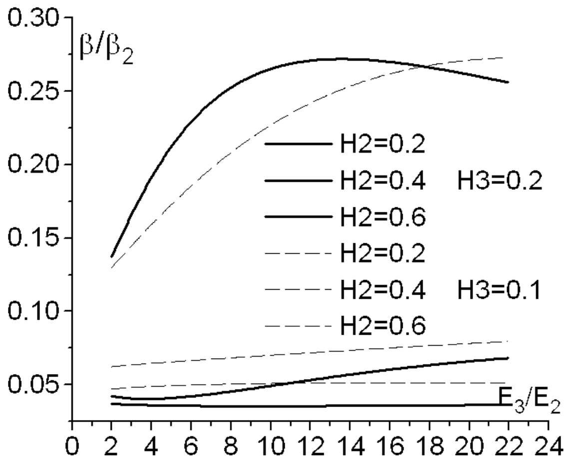


***Fig. 2 a. Loss factors in a symmetric five-layered plate with a constrained damping layers (layer 2)***

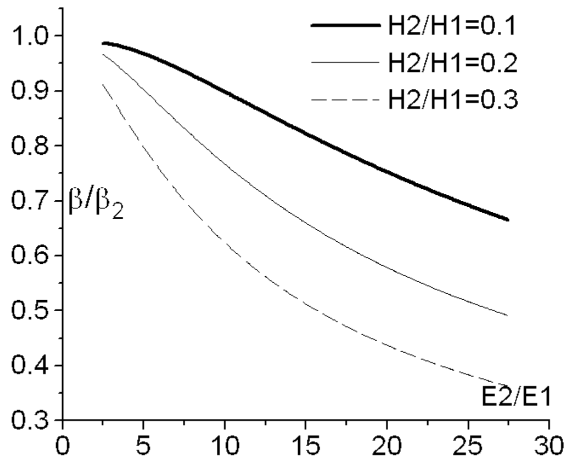


***Fig. 2 b. Loss factors in a three-layered plate with rigid faceplates and a damping core***

In Fig. 2 b, loss factors are presented for a three-layered beam with damping inner layers. Comparison is made with theory [8].

## 2. Theoretical model for experimental system

Figure 5 shows the experimental set up for the damping measurement in the composite layer sandwich beam system.

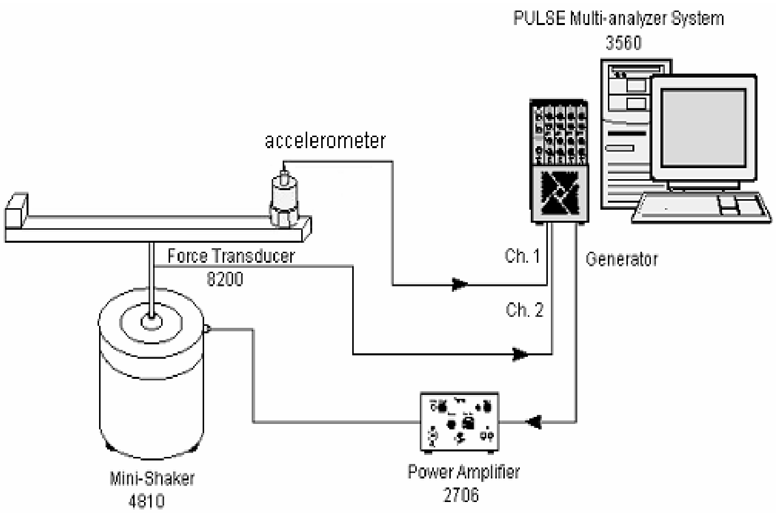


***Fig. 3. Experimental set up for damping measurements of sandwich beam***

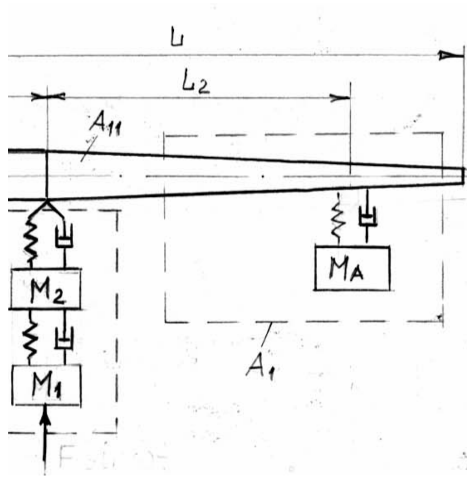


***Fig 4. The theoretical model for damping measurements of sandwich beam***

Figure 4 shows the theoretical model (one-half of the symmetrical one).

Let us consider a method **[**4**]** for the model. Divide the set o f elements $A_i$ of the mechanical structure into two subsets: 1) subset of joints elements $A_i^n$, and 2) subset of continuer elements $A_i^c$. For each of continuous element, consider a system of modal representation **[**4-7**]**

$$x_i\left(t,X\right)=\begin{bmatrix} q_{x1i}(t)\varphi_{x1i}(X) \\ \dots \\ q_{xni}(t)\varphi_{xni}(X)\end{bmatrix},\ y_i\left(t,X\right)=\begin{bmatrix} q_{y1i}(t)\varphi_{y1i}(X) \\ \dots \\ q_{yni}(t)\varphi_{yni}(X)\end{bmatrix},\ z_i\left(t,X\right)=\begin{bmatrix} q_{z1i}(t)\varphi_{z1i}(X) \\ \dots \\ q_{zni}(t)\varphi z_{zni}(X)\end{bmatrix}. \quad (2)$$

Let us define the kinematics and the elastic energy variations. In continuous elements:

$$\delta U_i^c=(K_i^{uc}\cdot q_i^c)^T\cdot\delta q_i^c,\ \ \delta K_i^c=(M_i^{kc}\cdot q_i^c)^T\cdot\delta q_i^c,$$

In discrete system elements

$$\delta U_i^n=(K_i^{un}\cdot q_i^c)^T\cdot\delta q_i^n,\ \ \delta K_i^n=(M_i^{kn}\cdot q_i^c)^T\cdot\delta q_i^n.$$

From the Hamilton-Ostrogradsky law we obtain, by summing terms collection near to the independent time functions variations $\delta q_i^c$ та $\delta q_i^n$,

$$(M\cdot\ddot{q}+K\cdot q)\cdot\delta q=0, \quad (3)$$

where

$$q=\left[q_{x11},q_{x12},\dots,q_{znn}\right]^T$$

*M* is the inertia matrix of, *K* – the stiffness matrix. It is important for the matrix *M* to be diagonal, which is well known, so that algorithms for solving Eq.(3) may be used. Another case in the case for more complicated joint elements in which the matrix *M* is not diagonal. But in **[**5-7**]** are some numerical matrix method introduced, are so that only few operations on matrix *M* the small order sub-matrix are needed. As is common with numerical methods such as FEM, the convolution of the whole matrix, is not needed.

## 3. Tymoshenko-beam model for laminated beam

Consider now for continuous element, which is in our case the Tymoshenko-beam model. The equivalent coefficient for such a beam should be derived near by some numerical experiments based on the theories **[**8**]**. As is well known, the kinematics assumptions are

$$U(X,Y,Z) = U_0(X) + \gamma(X)\cdot Z,\ \ W(X,Y,Z) = W_0(X). \tag{4}$$

From the Hamilton-Ostrogradsky law we obtain

$$\begin{aligned}&\int_0^L\left(EI\frac{\partial\gamma}{\partial x}\delta\frac{\partial\gamma}{\partial x} + GF\left(\gamma + \frac{\partial W}{\partial x}\right)\delta\gamma + \rho I\frac{\partial^2\gamma}{\partial t^2}\delta\gamma + \right.\\ &\left. +GF\left(\gamma + \frac{\partial W}{\partial x}\right)\delta\frac{\partial W}{\partial x} + \rho F\frac{\partial^2 W}{\partial t^2}\delta W\right)dx = F\end{aligned}\ , \tag{5}$$

where $F$ is an external force:

$$F = E_k(x)\cdot(W(0) - W_0)\delta W(0)\cdot + \left(I_m\cdot\frac{\partial^2\gamma(x_m)}{\partial t^2}\right)\delta\gamma(x_m) + \left(m_m\cdot\frac{\partial^2 W(x_m)}{\partial t^2}\right)\delta W(x_m) \tag{6}$$

Here $E$ elastic coefficient of beam joint and $I_m$, $m_m$ coefficients of the angular inertia and accelerometer mass. For coordinate functions we may use the power functions

$$W = q_i^w(t)\cdot x^i,\ \ \gamma = q_i^\gamma\cdot x^{(i-1)}. \tag{7}$$

By substitution of Eq. (7) into Eq. (5) we obtain

$$M_\gamma\frac{d^2\vec{\gamma}}{d\cdot t^2} = K_\gamma^\gamma\cdot\vec{\gamma} + K_w^\gamma\cdot\vec{w};\ \ M_w\frac{d^2\vec{w}}{d\cdot t^2} = K_w^\gamma\cdot\vec{\gamma} + K_w^w\cdot\vec{w} + \vec{j}., \tag{8}$$

systems which are ordinary differential equations. Here

$$\vec{\gamma} = (\gamma_1,\gamma_2,...,\gamma_n)^T;\ \ \vec{w} = (w_1,w_2,...,w_n)^T.$$

Frequency response functions (FRF) are presented in Fig. 5 for the various relations *E/G*

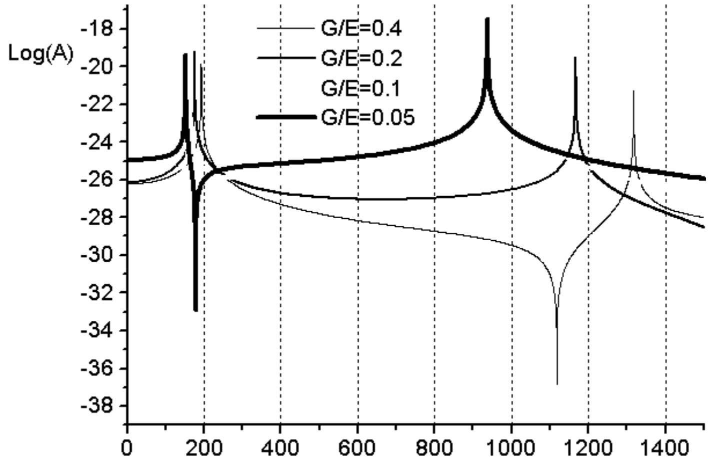


***Fig. 5. FRF of beams with various module relations***

Only a few vibration modes are needed for the determination of dynamic properties.

For the theoretical determination of GF and EI the exact analytical solution for the sandwich plate bending may be used **[5]**

$$u = \frac{Ax^2 z}{2C_x} + A\int_0^z \left( \int_0^z \frac{\alpha_2 z}{C_x} dz + \frac{\tau}{G} \right) dz - \Phi z\,, \quad w = -\frac{Ax^3}{6C_x} - Ax\int_0^z \frac{\alpha_2 z}{C_x} dz + \Phi x\,, \tag{9}$$

$$\tau = -A\int_z^H (1-\alpha_1\alpha_2) z dz\,, \quad \Phi = \frac{A}{H}\int_0^H dz \left( \int_0^z \frac{\alpha_2 z}{C_x} dz + \frac{\tau}{G} \right) dz\,. \tag{10}$$

Let us suppose $Q = 1$. Than

$$Q = \int_{-H}^{H} \tau dz = 1 \;\Rightarrow\; A = \left( \int_{-H}^{H} \left( \int_z^H (1-\alpha_1\alpha_2) z dz \right) dz \right)^{-1}. \tag{11}$$

Thus, by comparison with a uniform Tymoshenko beam of the same cross section, we may write an equation for the bending and transverse rigidity of a uniform equivalent beam

$$EI_T = M\Big/\frac{d\gamma}{dx} = \int_{-H}^{H} Z^2 \cdot \left( C_{XX} - C_{ZZ} C_{XZ} / C_{XX} \right) dZ\,, \frac{1}{2HG_T} = \int_{-H}^{H} \frac{\tau^2}{G} dz\,. \tag{12}$$

## 4. Experimental design

In the experiment the beam was excited with white noise by a shaker mounted at the middle. The excitation and responses were measured by a force transducer and an accelerometer, respectively. The experimental Bruel&Kjaer PULSE system analyzes the signals using dual FFT [9]. To balance the mass of the accelerometer loaded at one end of the beam, we added an identical mass at the other end. The density of the sandwich material was 278 kg/m$^3$ and the mass of the beam A was 27.33 g. For such a light structure a general purpose accelerometer is not applicable, because the effect of mass loading is significant [9,10]. The B&K PULSE system was used to analyze the signals with the Dual FFT mode and the damping ratio was determined directly.

Figure 6 compares the receptance frequency response functions of beams with single and double-layer face sheets. Double-layer face sheets add 13 % more mass to the beams. Figure 6 shows that the vibration properties are similar to that on Fig. 5.

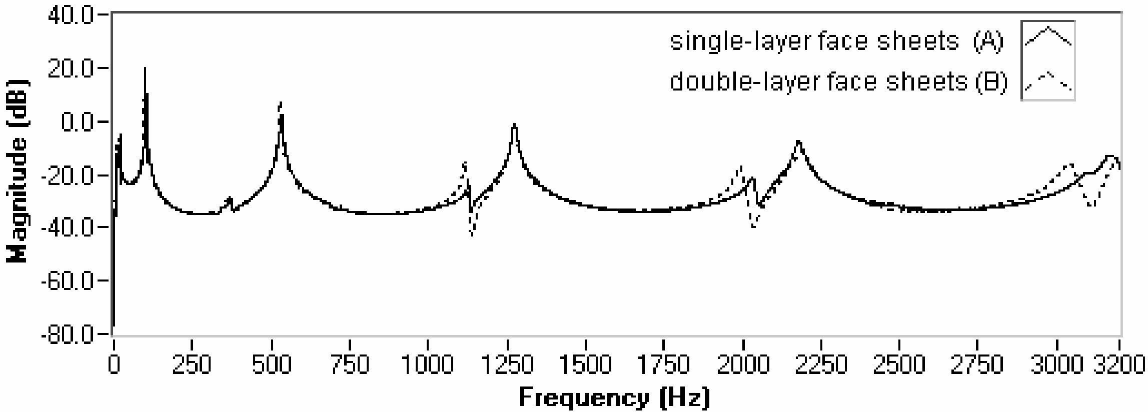


***Fig. 6. Receptance FRFs of beams A and B***

## Conclusions

The theoretical models for bounds of stiffness and damping of laminated structure have been developed. Several theoretical models for vibration test experiments have been reviewed. They may be used for FRF and damping properties of three-layered beams and plates with elastic honeycomb cores. and dynamical problem of laminated plates and shells. From the Tymoshenko-beam theory the discrete-continue models are obtained for vibration test set. With the small number of parameters studied it predicts dynamic behaviour of tested beam. Next, using the Tymoshenko model for layered beam, higher order modelling, not only damping by shear strain in the core may be discussed, but also the damping through layer normal and curving deformation. This is important for middle and higher frequency analysis of damping properties of sandwich structures.

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